

This is a five-page undergraduate paper on “Mathematics in the History of Culture.” It examines the influence of evolution on the derivation of mathematical concepts, finding that counting is not universal—it is a specific adaptation of quantification, as measuring also is. Set operations, on the other hand, appear to be universal.

## The Universality of Number: Don't Count on It

This paper proposes that the universality of mathematical concepts depends on whether they are derived from behaviours that are the unavoidable requirements of the principles of natural selection or they constitute adaptations driven by specific interactions with the environment. The former is exemplified by set operations, the latter by counting.

Let us accept Devlin's requirement that true linguistic and mathematical ability entail the capacity to think off-line. Then we would say that seals, herding smaller groups of fish into a larger school in order to facilitate a good catch, does not constitute mathematical ability. Even though the seals demonstrate an understanding of the ideas “greater than” and “less than” as well the practical application of addition, do not exhibit off-line mathematical thinking. Since off-line thinking involves the ability to employ abstractions at a conscious level, it presupposes key evolutionary roles for society and language. Even an appeal to processes of exaptation would be hard pressed to provide plausible alternative selective pressures to explain the emergence of the abstractions required for linguistic ability. Mathematical ability, then, seems to presuppose society and language.

Whether or not an intelligent creature has developed a concept of the individual in relation to a social group, it would necessarily have the ability to distinguish self from others. Such creatures would be capable of distinguishing classes of things—food and non-food, for example. They would also be able to distinguish self from the contents of

the environment—indeed from all others, including those of its own species. For survival is grounded in competition, and relies on behaviours based on such distinctions. As Lakoff and Nunez argue, mathematics is grounded in behaviour. Fundamental concepts, therefore, include classification, as well as inclusion and exclusion. Any intelligent creature would have to be able to construct sets that include, for example, others but not food. This suggests that carrying out operations with sets, at least at a basic level, constitutes a fundamental mathematical ability that is universal.

However, such concepts and the ability to carry out operations on them do not necessarily lead to the concept of counting. The origin of number would require some survival value, and establishing an adaptive advantage for the enumeration of classes or sets is not readily accomplished. However, the enumeration of discrete objects in the environment—as opposed to classes of objects—would directly impact survival, and it readily provides a plausible explanation for the emergence of counting as a mathematical concept. I have one fish; you have three: let's share.

Nevertheless, counting cannot be deemed a universal mathematical concept.

Consider the flagellate bacteria. Their mode of movement is chemotaxis, a mechanism that allows individual cells to move toward supplies of nutrients by detection of changes in chemical gradients. Moreover, they are able to discriminate among different kinds of nutrients and move preferentially toward those with higher energy values.

Imagine an intelligent species that interacts with its environment—a watery world—as flagellate bacteria do. Like bacteria, it feeds by absorption rather than ingestion, that is, rolling around in its food and absorbing microscopic nutrients into its body directly from

the environment. Supposing that this creature is not severely threatened by metazoan, predators, there would be little selection pressure for the development of sight organs. Without a need for the detection of discrete, macroscopic contents in its environment, there would be little advantage to vision, or its auditory equivalent, sonar. For such a species, quantity and measurement would supersede number and counting as concepts of importance to survival.

Of course, the survival of this hypothetical species requires the ability to discriminate among the nutrients in its environment. Like bacteria, it distinguishes among various kinds of nutrients—some higher in energy value and some lower. Based on gradients and energy values, it would have to decide whether a low concentration of high-energy nutrients is preferable to high concentration of low-energy nutrients and base its actions on the decision. Also like bacteria, it can detect the presence of a variety of threats in the environment, including the presence of toxins, heat, oxygen concentrations, pressure and the presence or absence of light. And like bacteria, it can respond in a way that takes into account all of the environmental factors, maximizing benefits and minimizing risks.

Without benefit of sight or sonar, such a socially and linguistically evolved creature would have developed the ability to think about its environment off-line. However, its mathematics is founded on quantity (amount) and direction—that is vectors. With the ability to model its environment in as a set of three-dimensional vector profiles for the various aspects of its world, this creature would be able not only to compare and contrast vector profiles but to perform addition (as well as other operations) on vectors rather than

numbers. It would appear to us that such a species possessed a highly sophisticated mathematical ability.

From these hypothetical constructions, it can be argued that quantification is a fundamental mathematical concept. In intelligent species that interact with their environment at a level of discrete macroscopic contents, aided by adaptations such organs for vision or sonar, quantification is realized in number and counting. In species that interact with their environment at a level of microscopic contents, assisted by adaptations for organs that sense molecular composition, quantification is realized in amount and measuring. Quantification is, therefore, the fundamental concept; measuring and counting are particular modes of that activity, adaptations selected for by the nature of the species' interaction with its environment.

It seems likely that a species' physical structure, nature of its sensory organs, and the way it interacts with its environment have the ability to affect some aspects of the mathematics it develops.

It might be argued that counting is an abstraction of a species' body structure. We have ten fingers, resulting in a number system in base 10. Presumably the eight-fingered inhabitants of television's *The Simpsons* would count in base 8. If we did not have digits but hooks or tentacles at our extremities, we might model our system on the limbs themselves, using base 4. An octopus, would likely count in base 8, like the Simpsons.

What if an intelligent species were characterized by an indeterminate number of limbs, varying from individual to individual? Such a species, if there were selection for interaction with discrete, macroscopic entities in its environment, might still develop counting, abstracted not from its own body structure, but from that of some element of

special significance in its environment, say a primary food source like a five-limbed star fish.

All of this is considered here to suggest that the manner of physical interaction with the environment is likely to be more important than particular body structure in determining a species as measurer or counter. (Of course, this is completely conjectural, as body structure evolves hand-in-hand, so to speak, with manner of physical interaction.) Just as selection pressure favours adaptations like visual or sonar sensory organs for species that interact predominantly with discrete macroscopic elements of the environment, so those sensory organs influence the mathematics that a species constructs.

However, as Sawyer points out, it is not only presence of particular sense organs, but their structure and the nature of the input derived from that influences the mathematics of a species. Consider the compound eyes of insects. While they are good at detecting motion, their ability in discriminating discrete objects is poor. Clearly such sight organs would not lend themselves to counting in intelligent species that possess them.

The ideas considered in this paper lead to the conjecture that some form of set operations are likely to be fundamental and universal mathematical concepts. Their apparent universality relies on the fact that they are grounded directly in the principles of natural selection. Counting, however, is not likely to be universal. It is one mode of the universal concept of quantification; measuring is another. Neither counting nor measuring is universal because each constitutes an adaptive form of quantification.